注意事項：

1. 本試卷共有 **9** 大題，每題配分標註於題號後，合計 100 分。
2. 請依序作答，且必須寫出**推導與計算過程**，否則不予計分。

[1] (10%) A prisoner is first asked to toss a fair coin. He is free if head appears; otherwise, a fair dice is then tossed to determine the number of days (which is the result of dice-tossing) he must stay in prison. Then, a fair coin and a fair dice (if necessary) are tossed over again until the prisoner is free.

(1) What is the probability that the prisoner stays in prison for 3 days before he is free? (5%)

(2) What is the expected number of days that the prisoner must stay in prison before he is free? (5%)

[2]. (10%) Let T_1 and T_2 be the repair times of machines 1 and 2 respectively, in which T_1 and T_2 are exponentially distributed with $E(T_1) = 40$ and $E(T_2) = 30$ minutes.

(1) Assume both machines start repairing simultaneously, what is the probability that machine 1 finishes repairing first? (5%)

(2) Assume machine 2 starts repairing **after** machine 1 has been repairing for 30 minutes, what is the probability that machine 1 finishes repairing first? (5%)

[3]. (10%) A dozen of canned cat food shows that the average weight is 98 grams. Assume $\sigma = 5$.

(1) Construct the 95% confidence interval for the average weight of canned cat food. (5%)

(2) How many cans of cat food are needed if the margin of error is 2 grams? (5%)

[4]. (10%) A survey report claimed that over 80% of teenagers spend more than 2 hours each day in playing **Pokémon Go**. A sample of 200 teenagers is asked and 140 of them do spend more than 2 hours each day.

(1) Construct the appropriate hypothesis test for the claim. (5%)

(2) Should the claim be accepted if $\alpha = 0.05$? (5%)

[5]. (10%) A process manager wants to determine if the processing times are different between methods A and B. Independent samples are taken from both methods, and the results are summarized as follows:

	Sample size n	Sample mean $\bar{x}$	Sample variance s^2
Method A	20	30	45
Method B	25	35	40

Conduct the appropriate hypotheses tests, and explain what the conclusion should be made? (10%)

Notes: In Problems 6 through 9,please use formal statistical methods to answer the following questions. For hypothesis testing, please define clearly the null hypothesis, alternative hypothesis, test statistic, and rejection rule.

[6]. (22%) Data on advertising expenditures ($1000s) and revenue ($1000s) for the CCUBA restaurant is as follows:

Advertising Expenditures	1	4	10	20	2	6	14
Revenue	19	44	52	54	32	40	53

(1) Let the advertising expenditures be the independent variable and the revenue be the response variable. Use the least squares method to develop a straight line approximation of the relationship between these two variables. (5%)

(2) Construct the ANOVA table and test whether revenue and advertising expenditures are related at a 0.05 level of significance. (10%)

(3) Draw a plot of residuals against the estimated values of the response variable. What conclusions can you draw from the residual analysis? Should this model be used, or should we look for a better one? Why? (7%)

[7]. (8%) Random samples from two normal populations produced the following statistics:

$s_1^2 = 350,\ n_1 = 21,\ s_2^2 = 700,\ n_2 = 16,$

where n_i and s_i^2, $i = 1,2$, are the sample sizes and sample variances, respectively. Perform a left-tailed test to determine whether there is sufficient evidence at a 0.05 level of significance to infer that the population variances differ? (8%)

[8]. (10%) The number of customer arrivals per hour at the CCUBA Company is assumed to have a Poisson distribution. Use $\alpha = 0.1$ (the level of significance) and the following data to test the assumption that the number of customer arrivals per hour has a Poisson distribution. (10%)

Number of arrivals per hour	0	1	2	3	4	5	6
Observed frequency	15	31	20	15	13	4	2

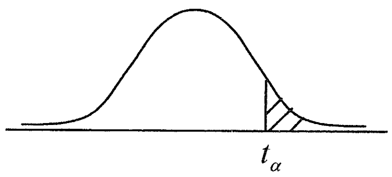
[9]. (10%) The data in the following table refer to four sample observations on two independent variables (X_1 and X_2) and a response variable (Y).

Case i	X_{1i}	X_{2i}	Y_i
1	2	6	23
2	8	9	83
3	6	8	63
4	10	10	103

Student A was asked to fit a multiple regression function to the data, and he returned in a short time with the fitted response function: $\hat{Y} = -7 + 9X_1 + 2X_2$. He was proud because the response function fit the data perfectly. According to the regression coefficients, does it imply that X_1 is the key independent variable and X_2 plays little role? Why? (10%)

Appendix: Distribution Tables

Table 1: *t* Distribution Critical Value Table



α value

Degrees of Freedom	0.3	0.2	0.1	0.05	0.025	0.01	0.005	0.001
1	0.727	1.376	3.078	6.314	12.706	31.821	63.657	318.309
2	0.617	1.061	1.886	2.920	4.303	6.965	9.925	22.327
3	0.584	0.978	1.638	2.353	3.182	4.541	5.841	10.215
4	0.569	0.941	1.533	2.132	2.776	3.747	4.604	7.173
5	0.559	0.920	1.476	2.015	2.571	3.365	4.032	5.893
6	0.553	0.906	1.440	1.943	2.447	3.143	3.707	5.208
7	0.549	0.896	1.415	1.895	2.365	2.998	3.499	4.785
8	0.546	0.889	1.397	1.860	2.306	2.896	3.355	4.501
9	0.543	0.883	1.383	1.833	2.262	2.821	3.250	4.297
10	0.542	0.879	1.372	1.812	2.228	2.764	3.169	4.144
11	0.540	0.876	1.363	1.796	2.201	2.718	3.106	4.025
12	0.539	0.873	1.356	1.782	2.179	2.681	3.055	3.930
13	0.538	0.870	1.350	1.771	2.160	2.650	3.012	3.852
14	0.537	0.868	1.345	1.761	2.145	2.624	2.977	3.787
15	0.536	0.866	1.341	1.753	2.131	2.602	2.947	3.733
16	0.535	0.865	1.337	1.746	2.120	2.583	2.921	3.686
17	0.534	0.863	1.333	1.740	2.110	2.567	2.898	3.646
18	0.534	0.862	1.330	1.734	2.101	2.552	2.878	3.610
19	0.533	0.861	1.328	1.729	2.093	2.539	2.861	3.579
20	0.533	0.860	1.325	1.725	2.086	2.528	2.845	3.552
21	0.532	0.859	1.323	1.721	2.080	2.518	2.831	3.527
22	0.532	0.858	1.321	1.717	2.074	2.508	2.819	3.505
23	0.532	0.858	1.319	1.714	2.069	2.500	2.807	3.485
24	0.531	0.857	1.318	1.711	2.064	2.492	2.797	3.467
25	0.531	0.856	1.316	1.708	2.060	2.485	2.787	3.450
26	0.531	0.856	1.315	1.706	2.056	2.479	2.779	3.435
27	0.531	0.855	1.314	1.703	2.052	2.473	2.771	3.421
28	0.530	0.855	1.313	1.701	2.048	2.467	2.763	3.408
29	0.530	0.854	1.311	1.699	2.045	2.462	2.756	3.396
30	0.530	0.854	1.310	1.697	2.042	2.457	2.750	3.385
40	0.529	0.851	1.303	1.684	2.021	2.423	2.704	3.307
50	0.528	0.849	1.299	1.676	2.009	2.403	2.678	3.261
60	0.527	0.848	1.296	1.671	2.000	2.390	2.660	3.232
70	0.527	0.847	1.294	1.667	1.994	2.381	2.648	3.211
80	0.526	0.846	1.292	1.664	1.990	2.374	2.639	3.195
90	0.526	0.846	1.291	1.662	1.987	2.368	2.632	3.183
100	0.526	0.845	1.290	1.660	1.984	2.364	2.626	3.174
120	0.526	0.845	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.092

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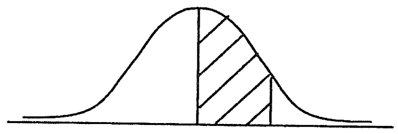
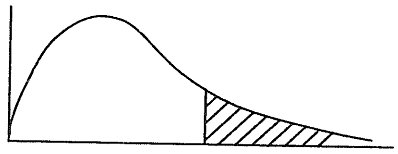
Table 2: The Standard Normal Distribution



Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995

Table 3: F Distribution Critical Value Table

(α value=0.05)

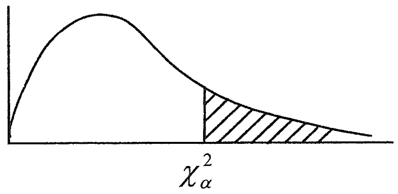


F

Denominator degrees of freedom (ddf) / Numerator degrees of freedom (ndf)

ddf \ ndf	1	2	3	4	5	6	7	8	9	10	12	15	20	40	60
1	161.400	199.500	215.700	224.600	230.200	234.000	236.800	238.900	240.500	241.900	243.900	246.000	248.000	251.100	252.200
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385	19.396	19.413	19.429	19.446	19.471	19.479
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786	8.745	8.703	8.660	8.594	8.572
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.858	5.803	5.717	5.688
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.678	4.619	4.558	4.464	4.431
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.938	3.874	3.774	3.740
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.511	3.445	3.340	3.304
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347	3.284	3.218	3.150	3.043	3.005
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	3.006	2.936	2.826	2.787
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.913	2.845	2.774	2.661	2.621
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.719	2.646	2.531	2.490
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.617	2.544	2.426	2.384
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.533	2.459	2.339	2.297
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.463	2.388	2.266	2.223
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.475	2.403	2.328	2.204	2.160
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.352	2.276	2.151	2.106
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.308	2.230	2.104	2.058
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.269	2.191	2.063	2.017
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.234	2.155	2.026	1.980
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.203	2.124	1.994	1.946
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321	2.250	2.176	2.096	1.965	1.916
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.226	2.151	2.071	1.938	1.889
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275	2.204	2.128	2.048	1.914	1.865
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255	2.183	2.108	2.027	1.892	1.842
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236	2.165	2.089	2.007	1.872	1.822
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220	2.148	2.072	1.990	1.853	1.803
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204	2.132	2.056	1.974	1.836	1.785
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190	2.118	2.041	1.959	1.820	1.769
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177	2.104	2.027	1.945	1.806	1.754
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165	2.092	2.015	1.932	1.792	1.740

Table 4: Chi-square Distribution Critical Value Table



α value

Degrees of Freedom	0995	0.99	0.975	0.95	0.9	0.8	0.2	0.1	0.05	0.025	0.01	0.005
1	0.000	0.000	0.001	0.004	0.016	0.064	1.642	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	0.446	3.219	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	1.005	4.642	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	1.649	5.989	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	2.343	7.289	9.236	11.07	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	3.070	8.558	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	3.822	9.803	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	4.594	11.030	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	5.380	12.242	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	6.179	13.442	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	6.989	14.631	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	7.807	15.812	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	8.634	16.985	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	9.467	18.151	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	10.307	19.311	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	11.152	20.465	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	12.002	21.615	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	12.857	22.760	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	13.716	23.900	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	14.578	25.038	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	15.445	26.171	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	16.314	27.301	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	17.187	28.429	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	18.062	29.553	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	18.940	30.675	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	19.820	31.795	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	20.703	32.912	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	21.588	34.027	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	22.475	35.139	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	23.364	36.250	40.256	43.773	46.979	50.892	53.672
31	14.458	15.655	17.539	19.281	21.434	24.255	37.359	41.422	44.985	48.232	52.191	55.003
32	15.134	16.362	18.291	20.072	22.271	25.148	38.466	42.585	46.194	49.480	53.486	56.328
33	15.815	17.074	19.047	20.867	23.110	26.042	39.572	43.745	47.400	50.725	54.776	57.648
34	16.501	17.789	19.806	21.664	23.952	26.938	40.676	44.903	48.602	51.966	56.061	58.964
35	17.192	18.509	20.569	22.465	24.797	27.836	41.778	46.059	49.802	53.203	57.342	60.275
40	20.707	22.164	24.433	26.509	29.051	32.345	47.269	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	41.449	58.164	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	50.641	68.972	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	59.898	79.715	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	69.207	90.405	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	78.558	101.054	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	87.945	111.667	118.498	124.342	129.561	135.807	140.169
110	75.550	78.458	82.867	86.792	91.471	97.362	122.250	129.385	135.480	140.917	147.414	151.948
120	83.852	86.923	91.573	95.705	100.624	106.806	132.806	140.233	146.567	152.211	158.950	163.648